

إجابات كتاب التمارين

التكامل المحدود

أجد قيمة كل من التكاملات الآتية:

$$\int (1510x - 2) dx$$

$$1510x - 2 dx = -10x - 1 \Big|_{15} = -10x \Big|_{15} = (-2) - (-10) = 8$$

$$\int (2x^3 - 4x + 5) dx$$

$$2x^3 - 4x + 5 dx = (12x^4 - 2x^2 + 5x) \Big|_{02} = 8 - 8 + 10 = 10$$

$$\int (14x^3 + 2x^2) dx$$

$$14x^3 + 2x^2 dx = \int 14(x^3 + 2x^2) dx = \int 14(x^5 + 2x^3) dx = (27x^7 + 45x^5) \Big|_{52} = (2567 + 1285) - (27 + 45) = 2547 + 1245 = 213835$$

$$\int (x - 3x)^2 dx$$

$$x - 3x)^2 dx = \int 36(x^2 - 6 + 9x^2) dx = \int 36(x^2 - 6 + 9x - 2) dx = (13x^3 - 6x) \Big|_{-9x-1} = (13x^3 - 6x - 9x) \Big|_{36} = (72 - 36 - 32) - (9 - 18 - 3) = 932$$

$$\int (x + 3| - 5) dx$$

$$x + 3| = \{-x - 3, x < -3; x + 3, x \geq -3\} \int 05(|x + 3| - 5) dx = \int 05(x + 3 - 5) dx = \int 05(x - 2) dx = (12x^2 - 2x) \Big|_{05} = (252 - 10) - (0 - 0) = 52$$

$$\int 06x(6 - x) dx$$

$$06x(6 - x) dx = \int 06(6x - x^2) dx = (3x^2 - 13x^3) \Big|_{06} = (108 - 2163) - (0) = 1083 = 36$$

$$\int (6x - 12x^4 + 3) dx$$

$$6x - 12x^4 + 3 dx = \int 12(6x - 12x - 4 + 3) dx = (3x^2 + 4x - 3 + 3x) \Big|_{12} = (3) \Big|_{12} = (12 + 12 + 6) - (3 + 4 + 3) = 172$$

$$\int_0^8 (2x-1) dx$$

$$\begin{aligned} 2x-1 &= \begin{cases} -2x+1, & x < 1 \\ 2x-1, & x \geq 1 \end{cases} \\ \int_0^8 (2x-1) dx &= \int_0^1 (-2x+1) dx + \int_1^8 (2x-1) dx \\ &= (-x^2+x)|_0^1 + (x^2-x)|_1^8 = (-1+1)-(0)+(49-7)-(1-12)=85 \end{aligned}$$

$$\int_{-4}^9 x dx$$

$$\begin{aligned} x &= \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases} \\ \int_{-4}^9 x dx &= \int_{-4}^0 -x dx + \int_0^9 x dx = -\frac{1}{2}x^2 \Big|_{-4}^0 + \frac{1}{2}x^2 \Big|_0^9 \\ &= (0) - (-92) + (81) - (0) = 252 \end{aligned}$$

$$\int_0^{12} (2x^2+x^3) dx$$

$$\begin{aligned} \int_0^{12} (2x^2+x^3) dx &= \int_0^{12} (2x^2+x^3) dx = \int_0^{12} (x^2+x^3) dx = \left(\frac{1}{3}x^3 + \frac{1}{4}x^4 \right) \Big|_0^{12} \\ &= \left(\frac{1}{3}(12)^3 + \frac{1}{4}(12)^4 \right) - \left(\frac{1}{3}(0)^3 + \frac{1}{4}(0)^4 \right) = 32 + 73 = 236 \end{aligned}$$

$$\int_0^4 (6x^2-4x) dx$$

$$\int_0^4 (6x^2-4x) dx = \left(2x^3 - 2x^2 \right) \Big|_0^4 = (128 - 32) - (0 - 0) = 96$$

$$\int_0^{10} (10x+1x^2) dx$$

$$\int_0^{10} (10x+1x^2) dx = 0$$

إذا كان $\int_0^6 g(x) dx = -2$, $\int_0^3 f(x) dx = 4$, $\int_0^2 f(x) dx = 5$, فأوجد كلاً مما يأتي:

$$\int_0^2 2f(x) dx$$

$$\int_0^2 2f(x) dx = 0$$

$$\int_0^{12} (f(x)-5) dx$$

$$\begin{aligned} \int_0^{12} (f(x)-5) dx &= \int_0^{12} f(x) dx - \int_0^{12} 5 dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx + \int_2^{12} f(x) dx - 5(12-0) \\ &= -4 + 5 + (-5 \times 10) = 1 + (-10) - (-5) = -4 \end{aligned}$$

$$\int_{-2}^{15} (2f(x)+5g(x)) dx$$

$$\int_{-2}^{15} (2f(x)+5g(x)) dx = -2 \int_{-2}^2 f(x) dx + 5 \int_{-2}^2 g(x) dx = -2(5) + 5(-2) = -20$$

$$(g(x)+2x)dx \quad (16) \quad 2-3 \int$$

$$(f(x)+g(x))dx \quad (17) \quad 2-3 \int$$

$$\int (f(x) + g(x)) dx = \int 2 - 3f(x) dx + \int 2 - 3g(x) dx = -5 + 2 = -3) 2 - 3 \int$$

$$(4f(x) - 3g(x))dx \quad (18) \quad 32 - \int$$

$$4\int_0^1 f(x) - 3g(x) dx = 4\int_0^1 f(x) dx - 3\int_0^1 g(x) dx = 4(5) - 3(-2) = 26$$

(19) إذا كان $f(x) = \begin{cases} x^2, & x < 2 \\ 8 - x, & x \geq 2 \end{cases}$ فأوجد قيمة $\int_{-3}^4 f(x) dx$.

$$36f(x)dx = \int -32f(x)dx + \int 26f(x)dx = \int -32x^2dx + \int 26(8-x)dx = (13x^3) - \int$$

$$|-32 + (8x - 12x^2)|26 = (83) - (-9) + (48 - 18) - (16 - 2) = 833$$

(20) سكان: أشارت دراسة إلى أن عدد السكان في إحدى القرى يتغير شهرياً بمعدل يمكن نمذجته بالاقتران: $P'(t) = 5 + 3t^{2/3}$ ، حيث t عدد الأشهر من الآن، و $P(t)$ عدد السكان، أجد مقدار الزيادة في عدد سكان القرية في الأشهر الثمانية القادمة.

$$P(t) = \int_0^8 (5 + 3t^2) dt = (5t + t^3) \Big|_0^8 = (40 + 512) - (0) = 552$$

(21) إذا كان: $\int_2^3 (x^2 - a) dx = 5$, فأوجد قيمة الثابت a .

$$\int (x^2 - a) dx = \left(\frac{1}{3}x^3 - ax \right) \Big|_2^3 = \left(\frac{1}{3}(3^3) - 3a \right) - \left(\frac{1}{3}(2^3) - 2a \right) = 9 - 3a - \left(\frac{8}{3} - 2a \right) = 9 - 3a - \frac{8}{3} + 2a = \frac{27}{3} - \frac{8}{3} - a = \frac{19}{3} - a$$