

أدرب وأحل المسائل

المماس والعمودي على المماس

أجد معادلة المماس لمنحنى كل اقتران ممّا يأتي عند النقطة المعطاة:

(1) $f(x) = x^3 - 6x + 3$, $(2, -1)$

$$f(x) = x^3 - 6x + 3, (2, -1), f(2) = -1, f'(x) = 3x^2 - 6, f'(2) = 12 - 6 = 6$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(2) = f'(2)(x - 2) \quad y - (-1) = 6(x - 2) \quad y + 1 = 6x - 12 \quad y = 6x - 13$$

(2) $f(x) = x^4 - 3x^3$, $(1, -2)$

$$f(x) = x^4 - 3x^3, (1, -2), f(1) = -2, f'(x) = 4x^3 - 9x^2, f'(1) = 4 - 9 = -5$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(1) = f'(1)(x - 1) \quad y - (-2) = -5(x - 1) \quad y + 2 = -5x + 5 \quad y = -5x + 3$$

(3) $f(x) = x(x^2 - 1)$, $(1, 0)$

$$f(x) = x(x^2 - 1), (1, 0), f(1) = 0, f'(x) = (x^2 - 1) + x(2x) = 3x^2 - 1, f'(1) = 3 - 1 = 2$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(1) = f'(1)(x - 1) \quad y - 0 = 2(x - 1) \quad y = 2x - 2$$

(4) $f(x) = x + 4x^2$, $(-4, -5)$

$$f(x) = x + 4x^2, (-4, -5), f(-4) = -5, f'(x) = 1 + 8x, f'(-4) = 1 - 32 = -31$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(-4) = f'(-4)(x - (-4)) \quad y - (-5) = -31(x + 4) \quad y + 5 = -31x - 124 \quad y = -31x - 129$$

(5) $f(x) = x + e^x$, $(0, 1)$

$$f(x) = x + e^x, (0, 1), f(0) = 1, f'(x) = 1 + e^x, f'(0) = 1 + e^0 = 2$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(0) = f'(0)(x - 0) \quad y - 1 = 2(x - 0) \quad y - 1 = 2x \quad y = 2x + 1$$

(6) $f(x) = \ln(x+e)$, $(0,1)$

$$f(x) = \ln(x+e) \text{ , } (0,1) \text{ , } f(0) = 1 \quad f'(x) = 1x + e \quad f'(0) = 10 + e = 1e$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(0) = f'(0)(x - 0) \quad y - 1 = 1e(x - 0) \quad y - 1 = 1ex \quad y = 1ex + 1$$

x أجد معادلة المماس لمنحنى كل اقتران ممّا يأتي عند قيمة المعطاة:

(7) $f(x) = x - 7$, $x = 16$

$$f(x) = x - 7 \text{ , } x = 16 \quad f(16) = 16 - 7 = 9 \rightarrow (16, 9) \quad f'(x) = 1 \quad f'(16) = 1 \quad y - 9 = 1(x - 16) \quad y = x - 7$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(16) = f'(16)(x - 16) \quad y - 9 = 1(x - 16) \quad y - 9 = x - 16 \quad y = x - 7$$

(8) $f(x) = (x-1)e^x$, $x = 1$

$$f(x) = (x-1)e^x \text{ , } x = 1 \quad f(1) = (1-1)e^1 = 0 \rightarrow (1, 0) \quad f'(x) = (x-1)e^x + e^x(1) = x e^x \quad f'(1) = 1e^1 = e$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(1) = f'(1)(x - 1) \quad y - 0 = e(x - 1) \quad y = ex - e$$

(9) $f(x) = x + 3x - 3$, $x = 4$

$$f(x) = x + 3x - 3 \text{ , } x = 4 \quad f(4) = 4 + 3(4) - 3 = 13 \quad f'(x) = (x-3)(1) - (x+3)(1) \quad f'(4) = -6(4-3) = -6$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \quad y - f(4) = f'(4)(x - 4) \quad y - 13 = -6(x - 4) \quad y = -6x + 24 + 13 = -6x + 37$$

$$y = -6x + 31$$

$$(10) f(x) = (\ln x)^2, x = e$$

$$f(x) = (\ln x)^2, x = e \Rightarrow f(e) = (\ln e)^2 = 1 \rightarrow (e, 1) \quad f'(x) = 2(\ln x)(1/x) \Rightarrow f'(e) = 2(\ln e)(1/e) = 2/e$$

معادلة المماس:

$$y - f(a) = f'(a)(x - a) \Rightarrow y - f(e) = f'(e)(x - e) \Rightarrow y - 1 = 2/e(x - e) \Rightarrow y - 1 = 2x/e - 2 \Rightarrow y = 2x/e + 1$$